Physics II
ISI B.Math
Back paper Exam : July 20, 2012

Total Marks: 75
Answer any five questions. EACH QUESTION carries 15 marks

1. One mole of a certain gas obeys the van der Waals equation of state $(P+\frac{a}{V^2})(V-b) = RT$ and the molar internal energy is given by $U = cT - \frac{a}{V}$ where a, b, c and R are constants. Calculate the molar heat capacities C_p and C_v.

Which of the two heat capacities is greater than the other and why?

2. State the 2nd law of thermodynamics as formulated by Kelvin-Planck and also by Clausius. Show that these two statements are equivalent.

3.A certain amount of ideal gas has volume V_1 and is at temperature T_1. The gas is then kept at constant pressure but is allowed to exchange thermal energy with a heat bath kept at a LOWER temperature T_2. Assume that the volume of the gas is V_2 after thermal equilibrium is reached. Show that the entropy change of the ideal gas is given by

$\triangle S = n \int_{T_1}^{T_2} C_v \frac{dT}{T} + nR(ln\frac{V_2}{V_1})$.

Is the change of entropy of the gas in this situation positive or negative?

Does your answer change if you consider the change in total entropy which includes entropy of the heat bath? Justify your answers.(Note: C_v is always positive)

4. Using the Helmholtz Free energy $F = U - TS$, derive the Maxwell's relation $\left(\frac{\partial S}{\partial V}\right)_T = \left(\frac{\partial P}{\partial T}\right)_V$.

Using the above or otherwise, for the phase transition of a two phase single substance system (like vapor and liquid water) derive the Clausius-Clapeyron equation

$\frac{dP}{dT} = \frac{L}{T\triangle V}$ where L is the specific latent heat, T is the temperature, and $\triangle V$ is the change in specific volume.

5. Define the partition function Z of a system with energy levels E_i (where i labels the states of the system) interacting with a heat bath at temperature T.

Show that the variance in the energy (otherwise known as the energy fluctuation) $\langle(\triangle E)^2\rangle$ is given by

$\langle(\triangle E)^2\rangle = \frac{\partial^2 lnZ}{\partial\beta^2}$ where $\beta = \frac{1}{kT}$.

6. A beam of monochromatic light of wavelength λ passes through two slits (at normal incidence) each of width b separated by a distance d. Show that the intensity of the light on a far away screen is given by
$I = I_0(\cos\beta)^2\left(\frac{\sin\alpha}{\alpha}\right)^2$ where $\alpha = \frac{\pi b\sin\theta}{\lambda}$ and $\beta = \frac{\pi d\sin\theta}{\lambda}$ and θ is the angle that denotes the location on the screen in the usual notation.